MATH 640A **Midterm Exam, Thursday 10 October** **Fall 2002**

Name: ______________________________ Staple this sheet to the front of your exam.

No books, notes, or calculators are allowed.
Show your work and **give reasons** for your conclusions.

1. (10 points) Find the Taylor's series for e^x about $x = 0$, and the error term when it is truncated after the x^3 term. If we use this truncated expansion to approximate e^1, can we be assured that the error is less than 0.1?

2. (5 points) Determine if $\sqrt{n-3} = o(n)$ as $n \to \infty$.

3. (10 points) Determine all sequences $\{x_i\}_{i=0}^{\infty}$ that satisfy the recurrence
$$x_{i+2} = \frac{x_{i+1}}{4} + \frac{x_i}{8}.$$
Is this recurrence stable?

4. (10 points) Estimate the relative error in computing $(a+b)/c$ for machine numbers a, b, and c. Assume a and b are positive and the unit roundoff is ϵ.

5. (10 points) Determine the condition number of the function $f(x) = \cos(x)$ at the point $x_0 = \pi/4$. If x_0 is rounded to $x_0(1+\delta)$ and we evaluate $f(x_0(1+\delta))$, how much relative error will we make?

6. (10 points) Suppose we know $f(3) < 0$ and $f(4) > 0$ for some continuous function f. Using bisection, how many iterations will it take to locate a root of f to within 10^{-5}? What could go wrong?

7. (15 points) Write well-commented pseudo-code to do one step of Newton's method to a polynomial using Horner's algorithm. (Do not worry about things like conserving memory.)

8. (30 points) Newton's method is based on linear approximations. In this problem we will construct and analyze a Quadratic Newton's Method (QNM). Suppose you want to find a root of $f(x)$. You make a guess x_0 and find $f(x_0) \neq 0$. You then compute $f'(x_0)$ and $f''(x_0)$. Given $\{x_0, f(x_0), f'(x_0), f''(x_0)\}$, we can construct a parabola that matches this data, find when it is zero, and use this as our next guess x_1. Continuing this process, we can construct an iterative algorithm.
   (a) Construct an explicit iteration formula for the QNM.
   (b) Does your formula have cancellation error that might lead to loss of significant digits? If so, rearrange the formula to minimize this problem.
   (c) Verify that if $f(r) = 0$ then r is a fixed point of your iteration.
   (d) Compute the order of convergence of the QNM.
   (e) Discuss the advantages and disadvantages of the QNM as compared to bisection, secant method, and Newton's method. Would you recommend it?