7.4

7.4 What are the dimensions of acceleration of gravity, density, dynamic viscosity, kinematic viscosity, specific weight, and speed of sound in (a) the *FLT* system, and (b) the *MLT* system? Compare your results with those given in Table 1.1 in Chapter 1.

$$g = \text{acceleration of gravity} = \frac{\text{velocity}}{\text{time}} \doteq \frac{L}{T^2}$$

$$\rho = \text{density} = \frac{\text{mass}}{\text{unit volume}} \doteq \frac{M}{L^3} \doteq \frac{FT^2}{L^4} \quad (\text{since } F \doteq MLT^{-2})$$

$$\mu = \text{dynamic viscosity} = \frac{\text{stress}}{\text{velocity gradient}} \doteq \frac{FL^{-2}}{T^{-1}} \doteq \frac{M}{LT}$$

$$\nu = \text{kinematic viscosity} = \frac{\text{dynamic viscosity}}{\text{density}} \doteq \frac{FL^{-2}T}{FT^2L^{-4}} \doteq \frac{L^2}{T}$$

$$\gamma = \text{specific weight} = \frac{\text{weight}}{\text{unit volume}} \doteq \frac{F}{L^3} \doteq \frac{(MLT^{-2})}{L^3} \doteq \frac{MT^{-2}}{L^2}$$

$$c = \text{speed of sound} = \frac{\text{length}}{\text{time}} = \frac{L}{T}$$

Thus,

(a) in the FLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{FL^{-4}T^2}}$

$\mu \doteq \underline{\underline{FL^{-2}T}}$

$\nu \doteq \underline{\underline{L^2T^{-1}}}$

$\gamma \doteq \underline{\underline{FL^{-3}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

(b) in the MLT system,

$g \doteq \underline{\underline{LT^{-2}}}$

$\rho \doteq \underline{\underline{ML^{-3}}}$

$\mu \doteq \underline{\underline{ML^{-1}T^{-1}}}$

$\nu \doteq \underline{\underline{L^2T^{-1}}}$

$\gamma \doteq \underline{\underline{ML^{-2}T^{-2}}}$

$c \doteq \underline{\underline{LT^{-1}}}$

7.14

7.14 Assume that the power, $\mathcal{P}$, required to drive a fan is a function of the fan diameter, D, the fluid density, ρ, the rotational speed, ω, and the flowrate, Q. Use D, ω, and ρ as repeating variables to determine a suitable set of pi terms.

$$\mathcal{P} = f(D, \rho, \omega, Q)$$

$\mathcal{P} \doteq FLT^{-1}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use D, ω, and ρ as repeating variables. Thus,

$$\Pi_1 = \mathcal{P}\, D^{a}\omega^{b}\rho^{c}$$

and

$$(FLT^{-1})(L)^{a}(T^{-1})^{b}(FL^{-4}T^{2})^{c} \doteq F^{0}L^{0}T^{0}$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-5$, $b=-3$, $c=-1$, and therefore

$$\Pi_1 = \frac{\mathcal{P}}{\rho D^{5}\omega^{3}}$$

Check dimensions using MLT system:

$$\frac{\mathcal{P}}{\rho D^{5}\omega^{3}} \doteq \frac{ML^{2}T^{-3}}{(ML^{-3})(L)^{5}(T^{-1})^{3}} \doteq M^{0}L^{0}T^{0} \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = Q\, D^{a}\omega^{b}\rho^{c}$$

$$(L^{3}T^{-1})(L)^{a}(T^{-1})^{b}(FL^{-4}T^{2})^{c} \doteq F^{0}L^{0}T^{0}$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$3 + a - 4c = 0 \qquad \text{(for } L\text{)}$$
$$-1 - b + 2c = 0 \qquad \text{(for } T\text{)}$$

(con't)

7.14 (con't)

It follows that $a=-3$, $b=-1$, $c=0$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \quad \therefore OK$$

Thus,

$$\underline{\underline{\frac{\mathcal{P}}{\rho D^5\omega^3} = \phi\left(\frac{Q}{D^3\omega}\right)}}$$

7.12

7.12 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $6-3=3$ dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p \, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F)$$
$$-2 + a + b - 2c = 0 \qquad \text{(for } L)$$
$$-b + c = 0 \qquad \text{(for } T)$$

It follows that $a=1$, $b=-1$, $c=-1$, and therefore

$$\Pi_1 = \frac{\Delta p \, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p \, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = D_2 \, D_1^a V^b \mu^c$$

$$L\,(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L)$$
$$-b + c = 0 \qquad \text{(for } T)$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.12 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^a V^b \mu^c$$

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$1+c=0 \qquad \text{(for } F\text{)}$$

$$-4+a+b-2c=0 \qquad \text{(for } L\text{)}$$

$$2-b+c=0 \qquad \text{(for } T\text{)}$$

It follows that $a=1,\ b=1,\ c=-1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \qquad \therefore \text{ok}$$

Thus,

$$\underline{\underline{\frac{\Delta p\, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1},\ \frac{\rho D_1 V}{\mu}\right)}}$$

From the continuity equation,

$$V\frac{\pi}{4}D_1^2 = V_s\frac{\pi}{4}D_2^2$$

where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should <u>not</u> be included as an independent variable.

7.16

7. The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $D \doteq L$ $\rho \doteq FL^{-4}T^{2}$ $\omega \doteq T^{-1}$ $Q \doteq L^{3}T^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^a \rho^b \omega^c$$

and

so that $(FL^{-2})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$

$1 + b = 0$ (for F)

$-2 + a - 4b = 0$ (for L)

$2b - c = 0$ (for T)

It follows that $a=-2$, $b=-1$, $c=-2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2\rho\omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \quad \therefore OK$$

For Π_2:

$$\Pi_2 = Q D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^2)^b(T^{-1})^c \doteq F^0L^0T^0$$

$b = 0$ (for F)

$3 + a - 4b = 0$ (for L)

$-1 + 2b - c = 0$ (for T)

It follows that $a=-3$, $b=0$, $c=-1$, and therefore

$$\Pi_2 = \frac{Q}{D^3\omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3\omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \quad \therefore OK$$

Thus,

$$\frac{\Delta p}{D^2\rho\omega^2} = \phi\left(\frac{Q}{D^3\omega}\right)$$

7.22

7.22 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0 L^0 T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0 L^0 T^0 \quad \therefore \text{OK}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K \frac{\ell}{D}$$

where K is some constant. It thus follows that

$$\underline{\underline{\Delta p \propto \frac{1}{D^2}}}$$

for a given velocity.

7.27

7.27 Assume that the drag, $\mathfrak{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathcal{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathcal{D} \doteq F \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1} \quad$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3$, etc.).

By inspection, for Π_1 (containing $\mathcal{D}$):

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

Thus,

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)$$

where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1}$, etc.

7.37

7.37 Air at 80 °F is to flow through a 2-ft pipe at an average velocity of 6 ft/s. What size pipe should be used to move water at 60 °F and average velocity of 3 ft/s if Reynolds number similarity is enforced?

For Reynolds number similarity,

$$Re_{air} = Re_{water} \text{ , or}$$

$$\left(\frac{VD}{\nu}\right)_{air} = \left(\frac{VD}{\nu}\right)_{water}$$

Thus,

$$D_{water} = \left(\frac{\nu_{water}}{\nu_{air}}\right)\left(\frac{V_{air}}{V_{water}}\right) D_{air}$$, where from Tables B.1 and B.2

$$\underset{60°F}{\nu_{water}} = 1.210 \times 10^{-5} \frac{ft^2}{s} \quad \text{and} \quad \underset{80°F}{\nu_{air}} = 1.69 \times 10^{-4} \frac{ft^2}{s}$$

Hence,

$$D_{water} = \left(\frac{1.210 \times 10^{-5} \frac{ft^2}{s}}{1.69 \times 10^{-4} \frac{ft^2}{s}}\right)\left(\frac{6 \frac{ft}{s}}{3 \frac{ft}{s}}\right)(2 \text{ ft}) = \underline{\underline{0.286 \text{ ft}}}$$

7.40

7.40 A model of a submarine, 1 : 15 scale, is to be tested at 180 ft/s in a wind tunnel with standard sea-level air, while the prototype will be operated in seawater. Determine the speed of the prototype to ensure Reynolds number similarity.

Let $(\ )_m$ and $(\ )_p$ denote model and prototype, respectively. Thus, $Re_m = Re_p$, or

$$\frac{V_m \ell_m}{\nu_m} = \frac{V_p \ell_p}{\nu_p}, \text{ where } \ell_m = \frac{1}{15}\ell_p$$

Hence,

$$V_m = \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{\ell_p}{\ell_m}\right)V_p = 15\left(\frac{\nu_m}{\nu_p}\right)V_p$$

Also,

$$\nu_m = 1.57\times10^{-4}\,\frac{ft^2}{s} \text{ and } \nu_p = 1.26\times10^{-5}\,\frac{ft^2}{s} \text{ so that}$$

$$V_m = 15\left(\frac{1.57\times10^{-4}\,ft^2/s}{1.26\times10^{-5}\,ft^2/s}\right)V_p = 187\,V_p$$

Thus,

$$V_p = \frac{V_m}{187} = \frac{180\,\frac{ft}{s}}{187} = \underline{\underline{0.963\,\frac{ft}{s}}}$$

7.42

7.42 The water velocity at a certain point along a 1 : 10 scale model of a dam spillway is 3 m/s. What is the corresponding prototype velocity if the model and prototype operate in accordance with Froude number similarity?

For Froude number similarity,

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

so that

$$V = \sqrt{\left(\frac{g}{g_m}\right)\left(\frac{\ell}{\ell_m}\right)}\; V_m$$

and with $g = g_m$, $\ell/\ell_m = 10$, $V_m = 3\ m/s$

$$V = \sqrt{10}\,\left(3\,\frac{m}{s}\right) = \underline{\underline{9.49\ \frac{m}{s}}}$$

7.47

7.47 (See Fluids in the News article "Modeling parachutes in a water tunnel," Section 7.8.1.) Flow characteristics for a 30-ft-diameter prototype parachute are to be determined by tests of a 1-ft-diameter model parachute in a water tunnel. Some data collected with the model parachute indicate a drag of 17 lb when the water velocity is 4 ft/s. Use the model data to predict the drag on the prototype parachute falling through air at 10 ft/s. Assume the drag to be a function of the velocity, V, the fluid density, ρ, and the parachute diameter, D.

$$\mathcal{D} = f(V, \rho, D)$$

$$\mathcal{D} \doteq F \qquad V \doteq LT^{-1} \qquad \rho \doteq FL^{-4}T^{2} \qquad D \doteq L$$

From the pi theorem, $4-3=1$ pi term required, and a dimensional analysis yields

$$\frac{\mathcal{D}}{\rho V^2 D^2} = C$$

where C is a constant. Thus, for similarity between model and prototype

$$\frac{\mathcal{D}}{\rho V^2 D^2} = \frac{\mathcal{D}_m}{\rho_m V_m^2 D_m^2}$$

so that

$$\mathcal{D} = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{D}{D_m}\right)^2 \mathcal{D}_m$$

$$= \left(\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right)\left(\frac{10\ \frac{ft}{s}}{4\ \frac{ft}{s}}\right)^2\left(\frac{30\ ft}{1\ ft}\right)^2(17\ lb)$$

$$= \underline{\underline{117\ lb}}$$

7.49

7.4 A 1/50 scale model is to be used in a towing tank to study the water motion near the bottom of a shallow channel as a large barge passes over. (See Video V7.16) Assume that the model is operated in accordance with the Froude number criteria for dynamic similitude. The prototype barge moves at a typical speed of 15 knots. (a) At what speed (in ft/s) should the model be towed? (b) Near the bottom of the model channel a small particle is found to move 0.15 ft in one second so that the fluid velocity at that point is approximately 0.15 ft/s. Determine the velocity at the corresponding point in the prototype channel.

(a) For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} \qquad (1)$$

Thus,

$$V_m = \sqrt{\frac{1}{50}}\,(15 \text{ knots}) = 2.12 \text{ knots}$$

From Table A.1 $\quad 1 \text{ knot} = \left(0.514 \frac{m}{s}\right)\left(3.281 \frac{\frac{ft}{s}}{\frac{m}{s}}\right) = 1.69 \frac{ft}{s}$

so that

$$V_m = (2.12 \text{ knots})\left(1.69 \frac{ft/s}{knot}\right) = \underline{\underline{3.58 \frac{ft}{s}}}$$

(b) Since from Eq. (1)

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\frac{1}{50}}$$

so that

$$V = \sqrt{50}\left(0.15 \frac{ft}{s}\right) = \underline{\underline{1.06 \frac{ft}{s}}}$$

7.54

7.54 A thin flat plate having a diameter of 0.3 ft is towed through a tank of oil (γ = 53 lb/ft^3) at a velocity of 5 ft/s. The plane of the plate is perpendicular to the direction of motion, and the plate is submerged so that wave action is negligible. Under these conditions the drag on the plate is 1.4 lb. If viscous effects are neglected, predict the drag on a geometrically similar, 2-ft-diameter plate that is towed with a velocity of 3 ft/s through water at 60 °F under conditions similar to those for the smaller plate.

If viscous and wave effects are neglected,

$$\mathcal{D} = f(d, \rho, V)$$

where: $\mathcal{D} \sim$ drag $\doteq F$, $d \sim$ plate diameter $\doteq L$, $\rho \sim$ fluid density $\doteq FL^{-4}T^2$, and $V \sim$ velocity $\doteq LT^{-1}$. From the pi theorem, 4-3 = 1 pi term required, and a dimensional analysis yields

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 d^2}$$

Since there is only one pi term

$$\frac{\mathcal{D}_m}{\rho_m V_m^2 d_m^2} = \frac{\mathcal{D}}{\rho V^2 d^2} = \text{constant}$$

where m refers to the smaller, 0.3-ft-diameter plate.

Thus,

$$\mathcal{D} = \frac{\rho}{\rho_m} \frac{V^2}{V_m^2} \frac{d^2}{d_m^2} \mathcal{D}_m \qquad (1)$$

From the data given:

$\rho = 1.94 \text{ slugs/ft}^3$; $d = 2 \text{ ft}$; $V = 3 \text{ ft/s}$

$\rho_m = \dfrac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}$; $d_m = 0.3 \text{ ft}$; $V_m = 5 \text{ ft/s}$; $\mathcal{D}_m = 1.4 \text{ lb}$

Therefore, from Eq. (1),

$$\mathcal{D} = \frac{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)}{\left(\frac{53 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)} \frac{\left(3 \frac{\text{ft}}{\text{s}}\right)^2}{\left(5 \frac{\text{ft}}{\text{s}}\right)^2} \frac{(2 \text{ ft})^2}{(0.3 \text{ ft})^2} (1.4 \text{ lb}) = \underline{\underline{26.4 \text{ lb}}}$$

7.75

7.75 River models are used to study many different types of flow situations. (See, for example, Video V7.12) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flowrate of 700 ft^3/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flowrate would the model operate?

For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}} \, \frac{A_m}{A}$$

Also, $$\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2$$

so that $$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \qquad (1)$$

Thus, $$\frac{\ell_m}{\ell} = 0.110$$

and for a prototype depth of 4 ft the corresponding model depth is

$$\ell_m = (0.110)(4\,ft) = \underline{\underline{0.440\,ft}}$$

The model flowrate is obtained from Eq. (1):

$$Q_m = \left(\frac{1}{250}\right)\left(700\,\frac{ft^3}{s}\right) = \underline{\underline{2.80\,\frac{ft^3}{s}}}$$