5.39

5.39 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.39. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

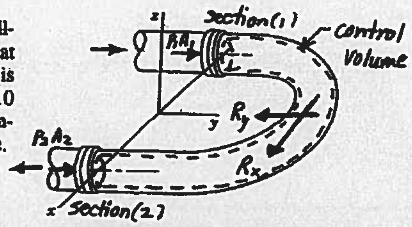


FIGURE P5.39

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.22) leads to

$$R_x = \underline{\underline{0}}$$

Application of the y-direction component of the linear momentum equation yields

$$-v_1 \rho v_1 A_1 - v_2 \rho v_2 A_2 = p_1 A_1 - R_y + p_2 A_2$$

or

$$R_y = \rho A_1 v_1 (v_1 + v_2) + p_1 A_1 + p_2 A_2$$

Thus

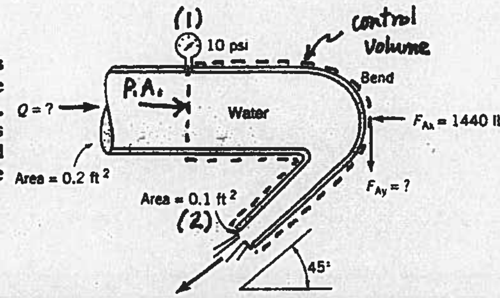
$$R_y = \left(999 \frac{kg}{m^3}\right) \frac{(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2} \left(15 \frac{m}{s}\right)\left(15\frac{m}{s} + 15\frac{m}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) + \frac{(210\ kPa)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPa}}\right)}$$

$$+ \frac{(165\ kPa)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPa}}\right)}$$

$$R_y = \underline{\underline{7420}}\ N$$

5.40

5.40 Water flows through a horizontal bend and discharges into the atmosphere as shown in Fig. P5.40. When the pressure gage reads 10 psi, the resultant x-direction anchoring force, F_{Ax}, in the horizontal plane required to hold the bend in place is shown on the figure. Determine the flowrate through the bend and the y direction anchoring force, F_{Ay}, required to hold the bend in place. The flow is not frictionless.



■ FIGURE P5.40

A control volume that contains the bend and the water within the bend between sections (1) and (2) as shown in the sketch above is used. Application of the x-direction component of the linear momentum equation yields

$$-u_1 \rho Q - V_2 \cos 45^\circ \rho Q = p_1 A_1 - F_{Ax} + \overset{0\text{ gage}}{p_2} A_2 \cos 45^\circ \qquad (1)$$

With

$$u_1 = \frac{Q}{A_1} \quad \text{and} \quad V_2 = \frac{Q}{A_2}$$

Eq. 1 becomes

$$-\frac{Q^2 \rho}{A_1} - \frac{Q^2 \rho \cos 45^\circ}{A_2} = p_1 A_1 - F_{Ax}$$

or for part (a)

$$Q = \sqrt{\frac{-p_1 A_1 + F_{Ax}}{\rho\left(\frac{\cos 45^\circ}{A_2} + \frac{1}{A_1}\right)}}$$

$$Q = \sqrt{\frac{-\left(10 \frac{\text{lb}}{\text{in.}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)\left(0.2\ \text{ft}^2\right) + 1440\ \text{lb}}{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(1 \frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right)\left(\frac{\cos 45^\circ}{0.1\ \text{ft}^2} + \frac{1}{0.2\ \text{ft}^2}\right)}}$$

$$Q = \underline{\underline{7.01}}\ \frac{\text{ft}^3}{\text{s}}$$

(con't)

5.40 (con't)

For part (b) we use the y-direction component of the linear momentum equation to get

$$F_{AY} = V_2 \sin 45^\circ \rho Q = \frac{Q}{A_2} \sin 45^\circ \rho Q$$

or

$$F_{AY} = \frac{Q^2}{A_2} \sin 45^\circ \rho$$

and

$$F_{AY} = \frac{\left(7.01 \frac{ft^3}{s}\right)^2}{(0.01\ ft^2)} \sin 45^\circ \left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right) = \underline{\underline{674\ lb}}$$

5.42

5.42 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.42 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

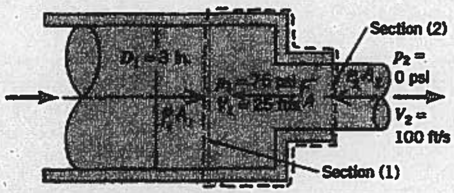


FIGURE P5.42

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = p_1 A_1 - F_A - p_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expressed as

$$\dot{m}(u_2 - u_1) = p_1 A_1 - F_A - p_2 A_2$$

or

$$F_A = p_1 A_1 - p_2 A_2 + \dot{m}(u_2 - u_1) = p_1 \frac{\pi}{4} D_1^2 - p_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi D_1^2}{4}(u_2 - u_1)$$

and

$$F_A = \left(75 \frac{lb}{in.^2}\right) \frac{\pi (3\,in.)^2}{4} - 0\,lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right) \frac{\pi (3\,in.)^2}{4 \left(144 \frac{in.^2}{ft^2}\right)} \left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352}}\ lb$$

5.49

5.49 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.49. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

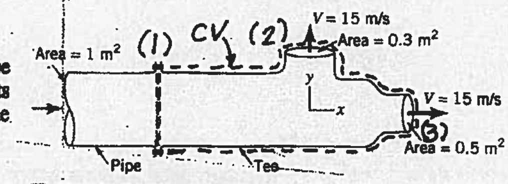


FIGURE P5.49

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3 \rho V_3 A_3 = p_1 A_1 - p_3 A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= (p_{1\,gage} + p_{atm})A_1 - (p_{3\,gage} + p_{atm})A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= p_{1\,gage} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $V_1 = \dfrac{A_2 V_2 + A_3 V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,\frac{m}{s}) + (0.5\,m^2)(15\,\frac{m}{s})}{1\,m^2} = 12\,\frac{m}{s}$

To estimate $p_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{p_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{p_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$p_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right)$$

$$p_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $F_x = \underline{\underline{72{,}000\,N}} \leftarrow$

For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2 \rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{67{,}400\,N} \uparrow = F_y$$

5.51

5.51 The hydraulic dredge shown in Fig. P5.51 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.

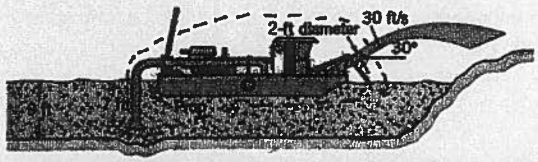


FIGURE P5.51

Using the control volume shown by the broken line in the sketch above we use the horizontal or x component of the linear momentum equation to get

$$F_x = \rho_2 A_2 V_2 V_{2x} = \rho_{H_2O}(sg)_2 \frac{\pi d_2^2}{4} V_2 V_2 \cos 30^\circ$$

where section 1 is where flow enters the control volume vertically and section 2 is where flow leaves the control volume at an angle of 30° from the horizontal direction. Note that there is no horizontal direction linear momentum flow at section 1.

$$F_x = \left(1.94 \frac{slugs}{ft^3}\right)(1.4) \frac{\pi (2\,ft)^2}{4} \left(30 \frac{ft}{s}\right)\left(30 \frac{ft}{s}\right) \cos 30^\circ \left(\frac{1\ lb}{\frac{ft\ slug}{s^2}}\right)$$

$$F_x = \underline{\underline{6650}}\ lb$$

5.60

5.60 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.60 estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.

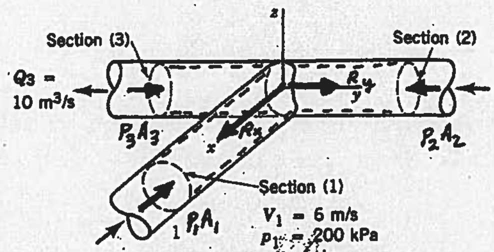


FIGURE P5.60

We can use the x and y components of the linear momentum equation (Eq. 5.22) to determine the x and y components of the reaction force exerted by the water on the tee. For the control volume containing water in the tee, Eq. 5.22 leads to

$$R_x = P_1A_1 + V_1\rho Q_1 = P_1\frac{\pi D_1^2}{4} + V_1\rho Q_1 \qquad (1)$$

and

$$R_y = P_2\frac{\pi D_2^2}{4} - P_3\frac{\pi D_3^2}{4} + V_2\rho Q_2 - V_3\rho Q_3 \qquad (2)$$

The reaction forces in Eqs. 1 and 2 are actually exerted by the tee on the water in the control volume. The reaction of the water on the tee is equal in magnitude but opposite in direction.

Conservation of mass (Eq. 5.4) leads to

$$Q_2 = Q_3 - Q_1 = Q_3 - V_1\frac{\pi D_1^2}{4} = 10\,\frac{m^3}{s} - \left(6\,\frac{m}{s}\right)\frac{\pi}{4}(1m)^2 = 5.288\,\frac{m^3}{s}$$

Also

$$Q_1 = V_1\frac{\pi D_1^2}{4} = \left(6\,\frac{m}{s}\right)\frac{\pi}{4}(1m)^2 = 4.712\,\frac{m^3}{s}$$

Further

$$V_2 = \frac{Q_2}{\frac{\pi D_2^2}{4}} = \frac{\left(5.288\,\frac{m^3}{s}\right)}{\frac{\pi(1m)^2}{4}} = 6.733\,\frac{m}{s}$$

and

$$V_3 = \frac{Q_3}{\frac{\pi D_3^2}{4}} = \frac{\left(10\,\frac{m^3}{s}\right)}{\frac{\pi(1m)^2}{4}} = 12.73\,\frac{m}{s}$$

(con't)

5.60 (con't)

Because the flow is incompressible and frictionless we assume that Bernoulli's equation (Eq. 5.74) is valid throughout the control volume. Thus

$$p_3 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_3^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(12.73\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_3 = 137\ kPa$$

Also

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(6.733\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_2 = 195.3\ kPa$$

With Eq. 1

$$R_x = \left(200{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(4.712\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) = 185{,}000\ N = 185\ kN$$

and the x-direction component of force exerted by the water on the tee is $\underline{\underline{-185\ kN}}$.

With Eq. 2

$$R_y = \left(195{,}300\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 - \left(137{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)5.2$$
$$+ \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(5.288\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)$$

or

$$R_y = -45{,}800\ N = -45.8\ kN$$

and the y-direction component of force exerted by the water on the tee is $\underline{\underline{+45.8}}\ kN$.

5.62

5.62 Determine the magnitude of the horizontal component of the anchoring force required to hold in place the sluice gate shown in Fig. 5.62. Compare this result with the size of the horizontal component of the anchoring force required to hold in place the sluice gate when it is closed and the depth of water upstream is 10 ft.

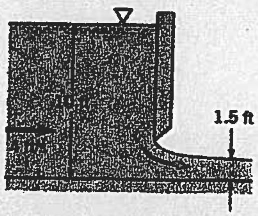


FIGURE P5.62

This analysis is similar to the one of Example 5.15. The control volumes of Fig. E 5.15 are appropriate for use in solving this problem. When the sluice gate is closed (see Figs. E5.15a and E5.15c) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 = \underline{\underline{3120}}\ \frac{lb}{ft}$$

When the sluice gate is open (see Figs. E5.15b and E5.15d) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_2^2 h$$

The exit velocity u_2 may be expressed in terms of the inlet velocity u_1 with the conservation of mass equation as follows

$$u_2 = u_1 \frac{H}{h}$$

Thus

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_1^2 \frac{H^2}{h}$$

Assuming F_f is negligibly small, we obtain

$$R_x = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 - \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(1.5\,ft)^2$$

$$+\left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2(10\,ft)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right) - \left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2\frac{(10\,ft)^2}{(1.5\,ft)}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$R_x = \underline{\underline{1290}}\ \frac{lb}{ft}$$

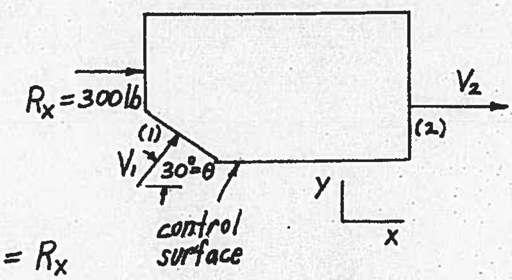
Thus it takes considerably less force to hold in place the sluice gate when it is opened as compared to when it is closed.

5.75

5.75 The thrust developed to propel the jet ski shown in Video V9.18 and Fig. P5.75 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.

FIGURE P5.75

For the control volume indicated the x-component of the momentum equation



$$\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \sum F_x \quad \text{becomes}$$

(1) $$(V_1 \cos 30°)\rho(-V_1)A_1 + V_2\,\rho(+V_2)A_2 = R_x$$

where we have assumed that $p=0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1 \cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25\,in.^2}{\frac{\pi}{4}(3.5\,in.)^2} V_1 = 2.60\,V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300\,lb}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(\frac{25}{144}\,ft^2\right)(2.60 - \cos 30°)}\right]^{1/2} = 22.7\,\frac{ft}{s}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144}\,ft^2\right)\left(22.7\,\frac{ft}{s}\right) = \underline{\underline{3.94\,\frac{ft^3}{s}}}$$

5.103

5.103 An incompressible liquid flows steadily along the pipe shown in Fig. P5.103. Determine the direction of flow and the head loss over the 6-m length of pipe.

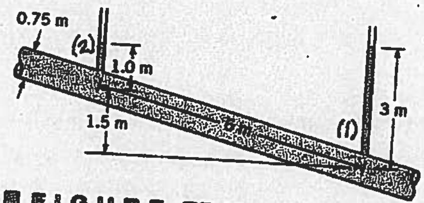


FIGURE P5.103

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_s}^{\,0} - h_\ell$$

Thus

$$h_\ell = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = \underline{\underline{0.5\,m}}$$

and since $h_\ell > 0$, the assumed direction of flow is correct.

<u>The flow is uphill.</u>

5.104

5.104 A siphon is used to draw water at 70°F from a large container as indicated in Fig. P5.104. The inside diameter of the siphon line is 1 in. and the pipe centerline rises 3 ft above the essentially constant water level in the tank. Show that by varying the length of the siphon below the water level, h, the rate of flow through the siphon can be changed. Assuming frictionless flow, determine the maximum flowrate possible through the siphon. The limiting condition is the occurrence of cavitation in the siphon. Will the actual maximum flow be more or less than the frictionless value? Explain.

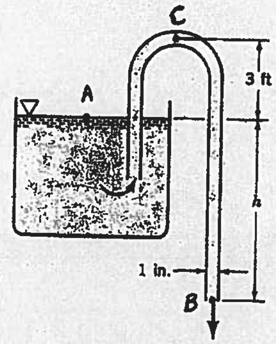


FIGURE P5.104

The flowrate, Q, can be determined with

$$Q = A_B V_B = \frac{\pi D_B^2}{4} V_B \qquad (1)$$

To obtain V_B we apply the energy equation (Eq. 5.82) between points A and B in the sketch above to obtain

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + gz_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + gz_A + \cancel{w_{\substack{shaft\\net\ in}}}^{0} - loss \qquad (2)$$

or

$$\frac{V_B^2}{2} = g(z_A - z_B) - loss$$

and

$$V_B = \sqrt{2\left[g(h) - loss\right]} \qquad (3)$$

With Eq. 3 we conclude that as h varies, so does V_B and thus Q. For no loss, the maximum flow will occur when the pressure at point C is just equal to the vapor pressure of water at 0°C. We apply the energy equation (Eq. 5.82) between points A and C to get

$$\frac{p_C}{\rho} + \frac{V_C^2}{2} + gz_C = \frac{p_A}{\rho} + \cancel{\frac{V_A^2}{2}}^{0} + gz_A + \cancel{w_{\substack{shaft\\net\ in}}}^{0} - \cancel{loss}^{0} \qquad (4)$$

Using absolute instead of gage pressures we obtain with Eq. 4

$$V_C = \sqrt{2g(z_A - z_C) + \frac{p_A - p_C}{\rho}}$$

or

$$V_C = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)(-3\,ft)\left(0.3048\,\frac{m}{ft}\right) + \frac{\left(101{,}000\,\frac{N}{m^2} - 1228\,\frac{N}{m^2}\right)}{\left(999.7\,\frac{kg}{m^3}\right)\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 9.048\,\frac{m}{s}$$

(con't)

5.104 (con't)

Since

$$Q = A_c V_c = \frac{\pi D_c^2}{4} V_c$$

we have for the maximum flowrate through the siphon,

$$Q = \frac{\pi (1 \text{ in.})^2}{4\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)} \left(0.3048 \frac{\text{m}}{\text{ft}}\right)^2 \left(9.048 \frac{\text{m}}{\text{s}}\right) = \underline{\underline{4.58 \times 10^{-3} \frac{\text{m}^3}{\text{s}}}}$$

With Eqs. 3 and 4 we conclude that any loss would act to lower the value of V in the siphon and thus make the actual maximum flowrate with friction less than the maximum flowrate without friction.

5.105

5.105 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.105. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

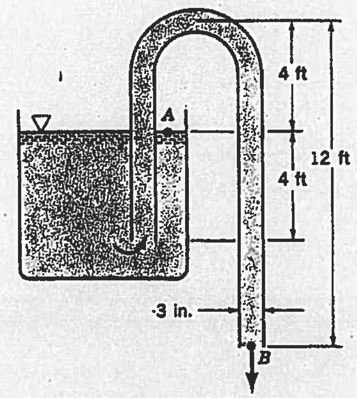


FIGURE P5.105

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + g z_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + w_{\substack{shaft \\ net\ in}} - \text{loss}$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8 \frac{V^2}{2}$$

Thus

$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{\left(32.2 \frac{ft}{s^2}\right)(8\ ft)}{0.9}} = 16.9 \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi (3\ in.)^2 \left(16.9 \frac{ft}{s}\right)}{4\left(144 \frac{in.^2}{ft^2}\right)} = \underline{\underline{0.830 \frac{ft^3}{s}}}$$

5.106

5.106 Water flows through a valve (see Fig. P5.106) at the rate of 1000 lbm/s. The pressure just upstream of the valve is 90 psi and the pressure drop across the valve is 5 psi. The inside diameters of the valve inlet and exit pipes are 12 and 24 in. If the flow through the valve occurs in a horizontal plane, determine the loss in available energy across the valve.

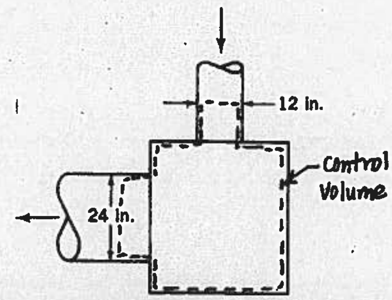


FIGURE P5.106

The control volume shown in the sketch above is used. We can use Eq. 5.79 to determine the loss in available energy associated with the incompressible, steady flow through this control volume. Thus

$$\text{loss} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2}$$

From the conservation of mass principle

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{\dot{m}}{\rho \pi \frac{D_1^2}{4}}$$

and

$$V_2 = \frac{\dot{m}}{\rho \pi \frac{D_2^2}{4}}$$

Thus

$$\text{loss} = \frac{p_1 - p_2}{\rho} + \frac{1}{2}\left(\frac{\dot{m}\,4}{\rho\pi}\right)^2\left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)$$

$$\text{loss} = \frac{\left(50\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{1.94\ \frac{slugs}{ft^3}} + \frac{1}{2}\left[\frac{\left(1000\ \frac{lbm}{s}\right)(4)}{\left(1.94\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\pi}\right]^2\left[\frac{\left(12\ \frac{in.}{ft}\right)^4}{(12\ in.)^4} - \frac{\left(12\ \frac{in.}{ft}\right)^4}{(24\ in.)^4}\right]\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$\text{loss} = \underline{\underline{5660}}\ \frac{ft \cdot lb}{slug}$$

5.108

5.108 For the 180° elbow and nozzle flow shown in Fig. P5.108 determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

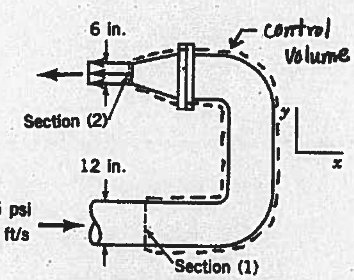


FIGURE P5.108

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.

From the conservation of mass principle we conclude that

$$V_2 = V_1 \frac{A_1}{A_2} = V_1\left(\frac{D_1^2}{D_2^2}\right)$$

Thus

$$_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

or

$$_1loss_2 = \frac{\left(15\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)} + \frac{\left(5\,\frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$_1loss_2 = \underline{\underline{926}}\ \frac{ft\cdot lb}{slug}$$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$_2loss_a = \frac{V_2^2}{2}$$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.

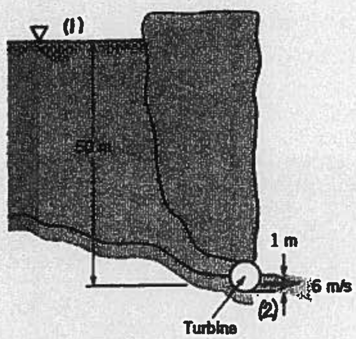
Thus

$$_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\,\frac{ft}{s}\right)^2\left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{2}$$

$$_2loss_A = \underline{\underline{200}}\ \frac{ft\cdot lb}{slug}$$

5.112

5.112 What is the maximum possible power output of the hydroelectric turbine shown in Fig. P5.112?



■ FIGURE P5.112

For flow from section (1) to section (2), Eq. 5.82 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + w_{\substack{shaft\\net\ in}} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$, $w_{\substack{shaft\\net\ in}} = -w_{\substack{shaft\\net\ out}}$ Eq. 1 can be expressed as

$$w_{\substack{shaft\\net\ out}} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when loss = 0.

Thus

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\, w_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\left[g(z_1 - z_2) - \frac{V_2^2}{2}\right]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = \left(999 \frac{kg}{m^3}\right)\left(6 \frac{m}{s}\right)\frac{\pi (1\ m)^2}{4} = 4710 \frac{kg}{s}$$

and

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \left(4710 \frac{kg}{s}\right)\left[\left(9.81 \frac{m}{s^2}\right)(50 m) - \frac{\left(6 \frac{m}{s}\right)^2}{2}\right]\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \underline{\underline{2.22 \times 10^6}} \frac{N \cdot m}{s} = \underline{\underline{2.22 \times 10^6}}\ W = \underline{\underline{2.22}}\ MW$$

5.113

5.113 Oil ($SG = 0.88$) flows in an inclined pipe at a rate of 5 ft^3/s as shown in Fig. P5.113. If the differential reading in the mercury manometer is 3 ft, calculate the power that the pump supplies to the oil if head losses are negligible.

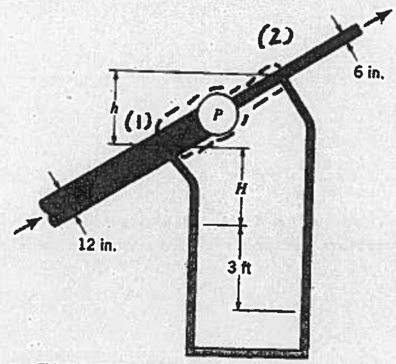


FIGURE P5.113

Using the control volume shown and the energy equation (Eq. 5.84) we get:

$$\frac{p_2}{\gamma_{oil}} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - h_L \quad (1)$$

The power supplied by the pump to the oil is, from Eq. 5.85:

$$\dot{W}_{\substack{shaft\\ net\ in}} = \gamma_{oil} Q h_s = SG_{oil}\, \gamma_{H_2O}\, Q h_s \quad (2)$$

Since $V = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}}$ we get

$$V_1 = \frac{5 \frac{ft^3}{s}}{\frac{\pi}{4}(1 ft)^2} = 6.37 \frac{ft}{s} \quad \text{and} \quad V_2 = \frac{5 \frac{ft^3}{s}}{\frac{\pi}{4}(\frac{1}{2} ft)^2} = 25.5 \frac{ft}{s}$$

From the manometer equation (see section 2.6) we get:

$$p_1 + \gamma_{oil} H + 3\gamma_{Hg} - (3 + H + h)\gamma_{oil} = p_2$$

Thus

$$\frac{p_1}{\gamma_{oil}} + H + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - (3 + H + h) = \frac{p_2}{\gamma_{oil}}$$

or

$$\frac{p_1}{\gamma_{oil}} + 3\frac{\gamma_{Hg}}{\gamma_{oil}} - 3 - h = \frac{p_2}{\gamma_{oil}} \quad (3)$$

Combining Eqs. (1) and (3) we get:

$$\frac{p_1}{\gamma_{oil}} + (3 ft)\frac{SG_{Hg}}{SG_{oil}} - (3 ft) - h + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma_{oil}} + \frac{V_1^2}{2g} + z_1 + h_s - \cancel{h_L}^{\,0}$$

or

$$h_s = z_2 - z_1 + 3 ft\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - h + \frac{V_2^2 - V_1^2}{2g} = (3 ft)\left(\frac{13.6}{0.88} - 1\right) + \frac{(25.5 \frac{ft}{s})^2 - (6.37 \frac{ft}{s})^2}{2(32.2 \frac{ft}{s^2})}$$

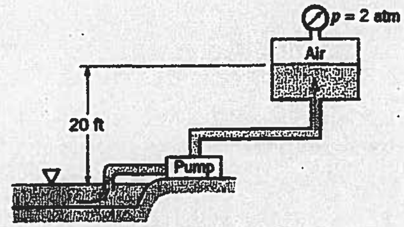
$$h_s = 52.9 ft$$

Finally from Eq. (2)

$$\dot{W}_{\substack{shaft\\ net\ in}} = (0.88)\left(62.4 \frac{lb}{ft^3}\right)\left(5 \frac{ft^3}{s}\right)(52.9 ft) = \underline{\underline{14{,}500 \frac{ft \cdot lb}{s}}}$$

5.116

5.116 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.116 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.



■ FIGURE P5.116

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_s-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g},$$ where $p_1=0$, $z_1=0$, $V_1=0$, and $z_2=20\text{ft}$.

Thus,

(1) $h_s=h_L+\frac{p_2}{\gamma}+z_2$

Also,

$$Q=\left[(1000\,gal)/(10\,min)\right]\left(\frac{1\,ft^3}{7.48\,gal}\right)\left(\frac{1\,min}{60\,s}\right)=0.223\,\frac{ft^3}{s}$$

so that

$$h_s=\frac{\dot W_s}{\gamma Q}=\frac{(3\,hp)(550\,\frac{ft\cdot lb/s}{hp})}{(62.4\,\frac{lb}{ft^3})(0.223\,\frac{ft^3}{s})}=119\,ft$$

(a) If $p_2=2\,atm=2(14.7\,\frac{lb}{in^2})(144\,in^2/ft^2)=4{,}230\,\frac{lb}{ft^2}$, then from Eq.(1)

$$h_s=h_L+\frac{4{,}230\,\frac{lb}{ft^2}}{(62.4\,\frac{lb}{ft^3})}+20\,ft=h_L+87.8\,ft$$

Thus, if

$h_L\le h_s-87.8\,ft=119\,ft-87.8\,ft=31.2\,ft$ the given pump will work for $p_2=2\,atm$.

(b) If $p_2=3\,atm=6{,}350\,\frac{lb}{ft^2}$, then

$$h_s=h_L+\frac{6{,}350\,\frac{lb}{ft^2}}{(62.4\,\frac{lb}{ft^3})}+20\,ft=h_L+122\,ft$$

Thus, if this pump is to work

$119\,ft=h_L+122\,ft$, or $h_L\le-3\,ft$

Since it is not possible to have $h_L<0$, the pump will not work for $p_2=3\,atm$.

5.131

5.131 Water flows steadily down the inclined pipe as indicated in Fig. P5.131. Determine the following: (a) The difference in pressure $p_1 - p_2$. (b) The loss per unit mass between sections (1) and (2). (c) The net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

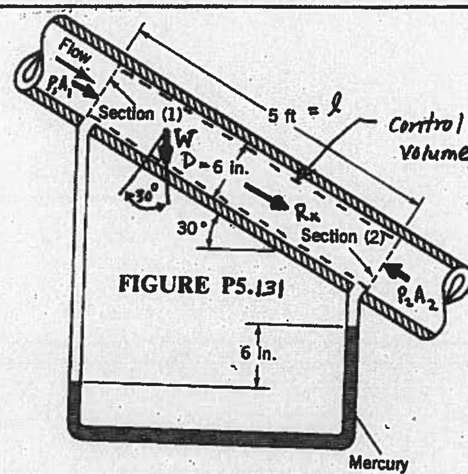


FIGURE P5.131

(a) The difference in pressure, $P_1 - P_2$, may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$P_1 - P_2 = -\gamma_{H_2O}\left[(5\,ft)\sin 30^\circ + \frac{(6\,in.)}{(12\,\frac{in.}{ft})}\right] + \gamma_{Hg}\frac{(6\,in.)}{(12\,\frac{in.}{ft})}$$

or

$$P_1 - P_2 = -\left(62.4\,\frac{lb}{ft^3}\right)\left[(5\,ft)\sin 30^\circ + (0.5\,ft)\right] + (13.6)\left(62.4\,\frac{lb}{ft^3}\right)(0.5\,ft) = 237\,\frac{lb}{ft^3}$$

and

$$P_1 - P_2 = 237\,\frac{lb}{ft^3}\,\frac{1}{\left(144\,\frac{in.^2}{ft^2}\right)} = \underline{\underline{1.65\,psi}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.79. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left(237\,\frac{lb}{ft^2}\right)\frac{1}{\left(1.94\,\frac{slugs}{ft^3}\right)}$$

$$+\left(32.2\,\frac{ft}{s^2}\right)(5\,ft)(\sin 30^\circ)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{203}}\,\frac{ft\cdot lb}{slug}$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.22). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4}(P_1 - P_2) - \gamma\frac{\pi D^2}{4}(\ell)\sin 30^\circ = -\frac{\pi D^2}{4}\left[(P_1 - P_2) + \gamma\ell\sin 30^\circ\right]$$

or

$$R_x = -\frac{\pi}{4}\left(\frac{6\,in.}{12\,\frac{in.}{ft}}\right)^2\left[237\,\frac{lb}{ft^2} + \left(62.4\,\frac{lb}{ft^3}\right)(5\,ft)\sin 30^\circ\right]$$

and

$R_x = -77.2\,lb$ = $\underline{\underline{77.2\,lb \text{ opposite to flow direction.}}}$

5.133

5.133 When fluid flows through an abrupt expansion as indicated in Fig. P5.133, the loss in available energy across the expansion, $loss_{ex}$, is often expressed as

$$loss_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

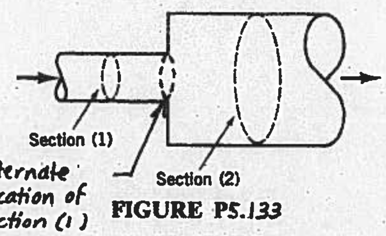


FIGURE P5.133

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + p_1 A_2 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, p_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

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From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$