5.11

5.11 A hydraulic jump (see Video V10.11) is in place downstream from a spillway as indicated in Fig. P5.11. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.

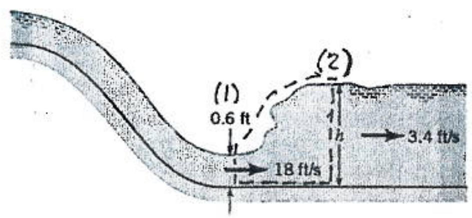


FIGURE P5.11

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

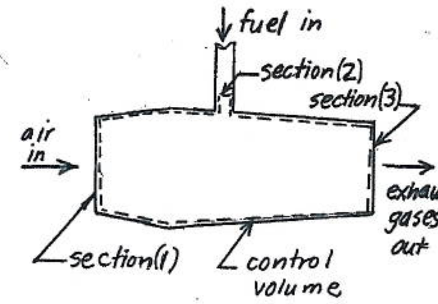
and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{(18\ \frac{ft}{s})(0.6\ ft)}{(3.4\ \frac{ft}{s})} = \underline{\underline{3.18\ ft}}$$

10 bi

5.14

5.14 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross-sectional area is 3.5 ft^2, estimate the density of the exhaust gases in lbm/ft^3.



For steady flow

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

$$\rho_3 A_3 \overline{V}_3 = \dot{m}_1 + \dot{m}_2$$

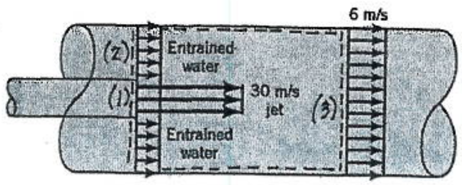
Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \overline{V}_3} = \frac{65 \frac{lbm}{s} + 0.60 \frac{lbm}{s}}{(3.5\ ft^2)(1500 \frac{ft}{s})}$$

$$\rho_3 = \underline{\underline{0.0125}} \frac{lbm}{ft^3}$$

5.18

5.18 A water jet pump (see Fig. P5.18) involves a jet cross-sectional area of 0.01 m^2, and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross-sectional area associated with the jet and entrained streams is 0.075 m^2. These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross-sectional area of 0.075 m^2. Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.



■ FIGURE P5.18

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

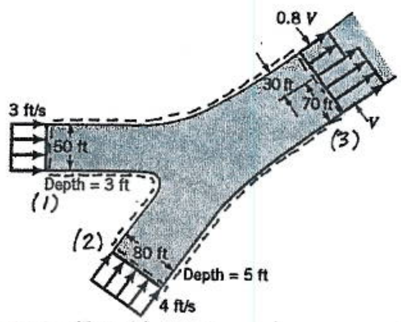
Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\,\frac{m}{s}\right)\left(0.075\,m^2\right) - \left(30\,\frac{m}{s}\right)\left(0.01\,m^2\right)\right]\left(1000\,\frac{liters}{m^3}\right)$$

$$Q_2 = 150\,\frac{liters}{s}$$

5.20

5.20 Two rivers merge to form a larger river as shown in Fig. P5.20. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of V.



■ FIGURE P5.20

Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V}\, 0.8V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50 ft)(3 ft)(3 \frac{ft}{s}) + (80 ft)(5 ft)(4 \frac{ft}{s})}{(30 ft)(6 ft)(0.8) + (70 ft)(6 ft)}$$

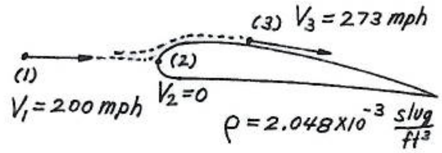
$$V = \underline{\underline{3.63}} \frac{ft}{s}$$

17 bi

3.19

3.19 When an airplane is flying 200 mph at 5000-ft altitude in a standard atmosphere, the air velocity at a certain point on the wing is 273 mph relative to the airplane. What suction pressure is developed on the wing at that point? What is the pressure at the leading edge (a stagnation point) of the wing?

(a) $$p + \frac{1}{2}\rho V^2 + z = \text{constant}$$



Thus, with $z_1 \approx z_2 \approx z_3$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^2 \text{ , but } p_1 = 0 \text{ so that}$$

$$p_3 = \frac{1}{2}\rho\left[V_1^2 - V_3^2\right] \quad \text{where } V_1 = 200\,mph\left(\frac{88\frac{ft}{s}}{60\,mph}\right) = 293\frac{ft}{s}$$

and

$$V_3 = 273\,mph\left(\frac{88\frac{ft}{s}}{60\,mph}\right) = 400\frac{ft}{s}$$

or

$$p_3 = \frac{1}{2}\left(2.05\times10^{-3}\frac{slugs}{ft^3}\right)\left[293^2 - 400^2\right]\frac{ft^2}{s^2}$$

$$= \underline{\underline{-76.0\frac{lb}{ft^2}}} \quad (gage)$$

(b) Also,

$$p_2 = \frac{1}{2}\rho V_1^2 = \frac{1}{2}\left(2.05\times10^{-3}\frac{slugs}{ft^3}\right)\left(293\frac{ft}{s}\right)^2 = \underline{\underline{88.0\frac{lb}{ft^2}}} \;(gage)$$

3.21

3.21 A loon is a diving bird equally at home "flying" in the air or water. What swimming velocity under water will produce a dynamic pressure equal to that when it flies in the air at 40 mph?

$$\frac{1}{2}\rho_{air}V_{air}^2 = \frac{1}{2}\rho_{H_2O}V_{H_2O}^2 \quad \text{or} \quad V_{H_2O} = \left[\frac{\rho_{air}}{\rho_{H_2O}}\right]^{1/2} V_{air}$$

Thus,

$$V_{H_2O} = \left[\frac{2.38\times10^{-3}\ \frac{slugs}{ft^3}}{1.94\ \frac{slugs}{ft^3}}\right](40\ mph) = \underline{\underline{1.40\ mph}}$$

2.1 bc

3.23

3.23 A person holds her hand out of an open car window while the car drives through still air at 65 mph. Under standard atmospheric conditions, what is the maximum pressure on her hand? What would be the maximum pressure if the "car" were an Indy 500 racer traveling 220 mph?

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with } z_1 = z_2$$

$$V_1 = 65\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 95.3\,\frac{ft}{s}$$

Thus, $\quad p_1 = 0,\ V_2 = 0$

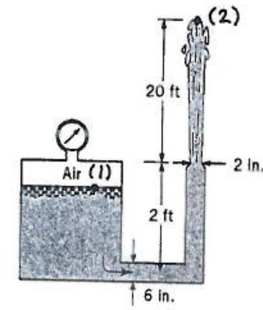
$$p_2 = \frac{\gamma}{2g}V_1^2 = \frac{1}{2}\rho V_1^2 \quad \text{or} \quad p_2 = \frac{1}{2}\left(2.38\times10^{-3}\,\frac{slug}{ft^3}\right)\left(95.3\,\frac{ft}{s}\right)^2 = \underline{\underline{10.8\,\frac{lb}{ft^2}}}$$

If $V_1 = 220\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 323\,\frac{ft}{s}$, then

$$p_2 = \frac{1}{2}\left(2.38\times10^{-3}\,\frac{slugs}{ft^3}\right)\left(323\,\frac{ft}{s}\right)^2 = \underline{\underline{124\,\frac{lb}{ft^2}}}$$

3.36

3.36 Water flows from a pressurized tank, through a 6-in.-diameter pipe, exits from a 2-in.-diameter nozzle, and rises 20 ft above the nozzle as shown in Fig. P3.36. Determine the pressure in the tank if the flow is steady, frictionless, and incompressible.



■ FIGURE P3.36

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 ,$$

where $V_1 = 0$, $V_2 = 0$, $z_1 = 2\,ft$, $z_2 = 22\,ft$, and $p_2 = 0$

Thus,

$$\frac{p_1}{\gamma} = z_2 - z_1$$

or

$$p_1 = \gamma(z_2 - z_1) = (62.4\,\frac{lb}{ft^3})(22\,ft - 2\,ft) = \underline{\underline{1248\,\frac{lb}{ft^2}}}$$

Note: The diameter of the pipe or nozzle are not needed.

3.40

3.40 For the pipe enlargement shown in Fig. P3.40 the pressures at sections (1) and (2) are 56.3 and 58.2 psi, respectively. Determine the weight flow rate (lb/s) of the gasoline in the pipe.

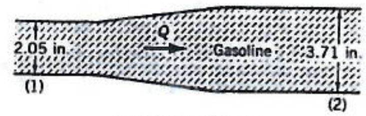


FIGURE P3.40

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } A_1V_1 = A_2V_2$$

or $V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1$

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{\left(\frac{D_1}{D_2}\right)^4 V_1^2}{2g}$$

or

$$V_1 = \sqrt{\frac{2g(p_2 - p_1)}{\gamma\left(1 - \left(\frac{D_1}{D_2}\right)^4\right)}} = \left[\frac{2\left(32.2\frac{ft}{s^2}\right)\left(58.2\frac{lb}{in.^2} - 56.3\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{42.5\frac{lb}{ft^3}\left(1 - \left(\frac{2.05\ in}{3.71\ in}\right)^4\right)}\right]^{1/2}$$

or

$$V_1 = 21.4\frac{ft}{s} \quad \text{and} \quad Q = A_1V_1 = \frac{\pi}{4}\left(\frac{2.05}{12}ft\right)^2\left(21.4\frac{ft}{s}\right) = 0.490\ \frac{ft^3}{s}$$

Thus,

$$\gamma Q = 42.5\frac{lb}{ft^3}\left(0.490\frac{ft^3}{s}\right) = \underline{\underline{20.8\ \frac{lb}{s}}}$$

3.45

3.45 Water (assumed inviscid and incompressible) flows steadily in the vertical variable-area pipe shown in Fig. P3.45. Determine the flowrate if the pressure in each of the gages reads 50 kPa..

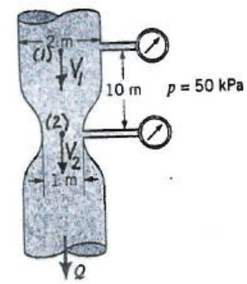


FIGURE P3.45

From the Bernoulli equation,

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2,$$

where $p_1 = p_2 = 50\ kPa$

Thus,

$$\frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = \gamma\left(z_1 - z_2\right) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$, or

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{\frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_1^2}\right) V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{1m}{2m}\right)^2 V_2 = \frac{1}{4} V_2$$

Hence, Eq. (1) becomes

$$\frac{1}{2}\rho\left[V_2^2 - \frac{1}{16}V_2^2\right] = \rho g\left(z_1 - z_2\right)$$

or

$$\frac{15}{16}V_2^2 = 2g\left(z_1 - z_2\right) = 2\left(9.81\frac{m}{s}\right)\left(10m\right)$$

or

$$V_2 = 14.5\frac{m}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4}(1m)^2\left(14.5\frac{m}{s}\right) = \underline{\underline{11.4\frac{m^3}{s}}}$$

39 bi

3.51

3.51 Water flows through the pipe contraction shown in Fig. P3.51. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

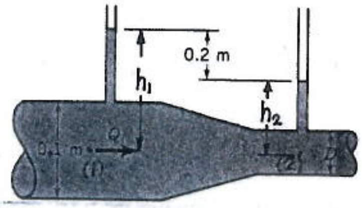


FIGURE P3.51

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \qquad \text{with } A_1V_1=A_2V_2$$

Thus, with $z_1=z_2$ or $V_2=\frac{\left(\frac{\pi}{4}D_1^2\right)}{\left(\frac{\pi}{4}D_2^2\right)}V_1=\left(\frac{0.1}{D}\right)^2V_1$

$$\frac{p_1-p_2}{\gamma}=\frac{V_2^2-V_1^2}{2g}=\frac{\left[\left(\frac{0.1}{D}\right)^4-1\right]V_1^2}{2g}$$

but

$p_1=\gamma h_1$ and $p_2=\gamma h_2$ so that $p_1-p_2=\gamma(h_1-h_2)=0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma}=\frac{\left[\left(\frac{0.1}{D}\right)^4-1\right]V_1^2}{2g} \quad \text{or} \quad V_1=\sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4-1\right]}}$$

and

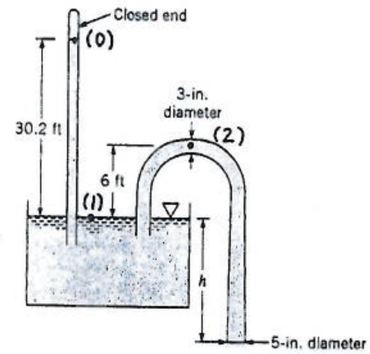
$$Q=A_1V_1=\frac{\pi}{4}(0.1)^2\sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4-1\right]}}$$

or

$$Q=\frac{0.0156\,D^2}{\sqrt{(0.1)^4-D^4}}\ \frac{m^3}{s} \quad \text{when } D\sim m$$

3.58

3.58 Water is siphoned from the tank shown in Fig. P3.58. The water barometer indicates a reading of 30.2 ft. Determine the maximum value of h allowed without cavitation occurring. Note that the pressure of the vapor in the closed end of the barometer equals the vapor pressure.



■ FIGURE P3.58

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ V_1 = 0,\ p_2 = p_{vapor}$$

Thus, $\qquad z_1 = 0,\ z_2 = 6\text{ ft}$

$$0 = \frac{p_{vapor}}{\gamma} + \frac{V_2^2}{2g} + 6\text{ ft}$$

but $p_0 + 30.2\text{ ft}\,\gamma = p_1$ or since $p_0 = p_{vapor}$, $\frac{p_{vapor}}{\gamma} = -30.2\text{ ft}$

Hence,

$$0 = -30.2\text{ ft} + \frac{V_2^2}{2g} + 6\text{ ft} \quad \text{or} \quad \frac{V_2^2}{2g} = 24.2\text{ ft} \quad \text{or} \quad V_2^2 = \left[2\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(24.2\text{ ft})\right]$$

Thus,

$$V_2 = 39.5\ \frac{\text{ft}}{\text{s}}$$

Since $V_3 A_3 = V_2 A_2$, $\quad V_3 = \frac{A_2}{A_3} V_2 = \frac{D_2^2}{D_3^2} V_2 = \left(\frac{3\text{ in.}}{5\text{ in.}}\right)^2 \left(39.5\ \frac{\text{ft}}{\text{s}}\right)$

or

$$V_3 = 14.2\ \frac{\text{ft}}{\text{s}}$$

However,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{or} \quad V_3 = \sqrt{2gh}$$

Thus,

$$14.2\ \frac{\text{ft}}{\text{s}} = \sqrt{2\left(32.2\frac{\text{ft}}{\text{s}^2}\right) h\text{ ft}} \quad \text{or} \quad \underline{\underline{h = 3.13\text{ ft}}}$$

3.61

3.61 Water is siphoned from the tank shown in Fig. P3.61. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.

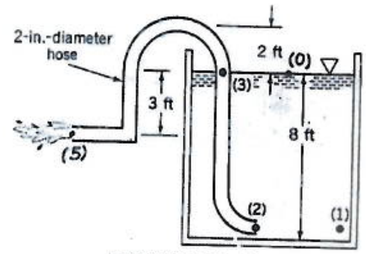


FIGURE P3.61

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i \quad \text{for } i = 1, \dots, 5 \qquad (1)$$

For $i = 5$ and $p_0 = 0$, $V_0 = 0$, $p_5 = 0$ this becomes

$$z_0 = \frac{V_5^2}{2g} + z_5 \quad \text{or} \quad V_5 = \sqrt{2g(z_0 - z_5)} = \sqrt{2(32.2\,\frac{ft}{s^2})(3\,ft)} = 13.9\,\frac{ft}{s}$$

Thus,

$$Q = A_5 V_5 = \frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2 (13.9\,\frac{ft}{s}) = \underline{\underline{0.303\,\frac{ft^3}{s}}}$$

From Eq.(1) with $i = 1$ and $V_1 = 0$, $p_1 = \gamma(z_0 - z_1)$

$$= (62.4\,\frac{lb}{ft^3})(8\,ft) = \underline{\underline{499\,\frac{lb}{ft^2}}}$$

From Eq.(1) with $i = 2$, $z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

where $A_2 V_2 = A_5 V_5$

Since $A_2 = A_5$ it follows that $V_2 = V_5$ or $\frac{V_2^2}{2g} = \frac{V_5^2}{2g} = z_0 - z_5$

Thus,

$$\frac{p_2}{\gamma} = z_0 - z_2 - \frac{V_2^2}{2g} = z_0 - z_2 - \frac{V_5^2}{2g} = z_0 - z_2 - (z_0 - z_5)$$

or

$$= z_5 - z_2$$

$$p_2 = \gamma(z_5 - z_2) = (62.4\,\frac{lb}{ft^3})(5\,ft) = \underline{\underline{312\,\frac{lb}{ft^2}}}$$

From Eq. (1) with $i = 3$, $z_0 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$

where $A_3 V_3 = A_5 V_5$

Since $A_3 = A_5$ it follows that $V_3 = V_5$ or $\frac{V_3^2}{2g} = \frac{V_5^2}{2g} = z_0 - z_5$

Thus,

$$\frac{p_3}{\gamma} = z_0 - z_3 - \frac{V_3^2}{2g} = z_0 - z_3 - \frac{V_5^2}{2g} = z_0 - z_3 - (z_0 - z_5)$$

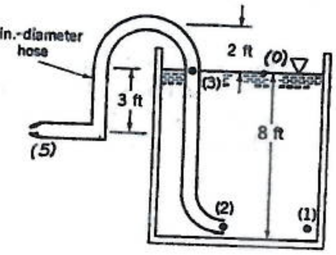
or

$$= z_5 - z_3$$

$$p_3 = \gamma(z_5 - z_3) = (62.4\,\frac{lb}{ft^3})(-3\,ft) = \underline{\underline{-187\,\frac{lb}{ft^2}}}$$

3.62

3.62 Redo Problem 3.61 if a 1-in.-diameter nozzle is placed at the end of the tube.



$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=\frac{p_i}{\gamma}+\frac{V_i^2}{2g}+z_i \quad \text{for } i=1,\dots,5 \qquad (1)$$

For $i=5$ and $p_0=0$, $V_0=0$, $p_5=0$ this becomes

$z_0=\frac{V_5^2}{2g}$ or $V_5=\sqrt{2g(z_0-z_5)}=\sqrt{2(32.2\,\frac{ft}{s^2})(3\,ft)}$

Thus, $\quad = 13.9\,\frac{ft}{s}$

$Q=A_5V_5=\frac{\pi}{4}\left(\frac{1}{12}\,ft\right)^2\left(13.9\,\frac{ft}{s}\right)=\underline{\underline{0.0758\,\frac{ft^3}{s}}}$

From Eq. (1) with $i=1$ and $V_1=0$, $p_1=\gamma(z_0-z_1)$

$=(62.4\,\frac{lb}{ft^3})(8\,ft)=\underline{\underline{499\,\frac{lb}{ft^2}}}$

From Eq. (1) with $i=2$, $z_0=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$

where $A_2V_2=A_5V_5$

Since $A_2=\left(\frac{D_2}{D_5}\right)^2A_5=\left(\frac{2}{1}\right)^2A_5=4A_5$ it follows that

$V_2=\frac{1}{4}V_5$ or $\frac{V_2^2}{2g}=\frac{1}{2g}\left(\frac{1}{4}V_5\right)^2=\frac{1}{16}\frac{V_5^2}{2g}=\frac{1}{16}(z_0-z_5)$

Thus,

$\frac{p_2}{\gamma}=z_0-z_2-\frac{V_2^2}{2g}=z_0-z_2-\frac{1}{16}(z_0-z_5)=8\,ft-\frac{1}{16}(3\,ft)$

$= 7.81\,ft$

or

$p_2=(62.4\,\frac{lb}{ft^3})(7.81\,ft)=\underline{\underline{488\,\frac{lb}{ft^2}}}$

From Eq. (1) with $i=3$, $z_0=\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3$

where $A_3V_3=A_5V_5$

or since $A_3=A_2$ then $V_3=V_2$ and $\frac{V_3^2}{2g}=\frac{V_2^2}{2g}=\frac{1}{16}(z_0-z_5)$

Thus,

$\frac{p_3}{\gamma}=z_0-z_3-\frac{V_3^2}{2g}=z_0-z_3-\frac{1}{16}(z_0-z_5)=-\frac{1}{16}(3\,ft)=-\frac{3}{16}\,ft$

or

$p_3=(62.4\,\frac{lb}{ft^3})(-\frac{3}{16}\,ft)=\underline{\underline{-11.7\,\frac{lb}{ft^2}}}$

3.67

3.67 Oil of specific gravity 0.83 flows in the pipe shown in Fig. P3.67. If viscous effects are neglected, what is the flowrate?

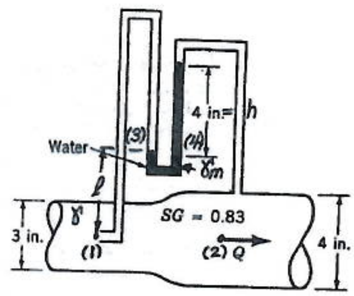


FIGURE P3.67

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_1 = 0$$

Thus,

$$\frac{V_2^2}{2g} = \frac{p_1 - p_2}{\gamma} \qquad (1)$$

but,

$$p_1 = p_3 + \gamma \ell = p_4 + \gamma \ell$$

and

$$p_2 = \gamma(\ell + h) - \gamma_m h + p_4$$

Thus,

$$p_1 - p_2 = (\gamma_m - \gamma)h \qquad (2)$$

Combine Eqs. (1) and (2) to obtain

$$V_2 = \sqrt{2g\left(\frac{p_1 - p_2}{\gamma}\right)} = \sqrt{2g\left(\frac{\gamma_m}{\gamma} - 1\right)h} = \sqrt{2\left(32.2\,\frac{ft}{s^2}\right)\left(\frac{62.4\,\frac{lb}{ft^3}}{0.83\left(62.4\,\frac{lb}{ft^3}\right)} - 1\right)\left(\frac{4}{12}\,ft\right)}$$

or

$$V_2 = 2.10\,\frac{ft}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{4}{12}\,ft\right)^2\left(2.10\,\frac{ft}{s}\right) = \underline{\underline{0.183\,\frac{ft^3}{s}}}$$

3.69

3.69 Determine the flowrate through the pipe in Fig. P3.69.

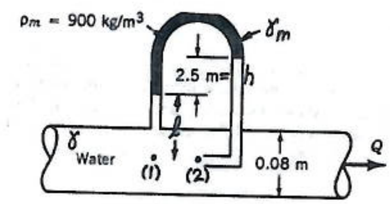


FIGURE P3.69

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } z_1 = z_2 \text{ and } V_2 = 0$$

Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} \quad \text{or} \quad V_1 = \sqrt{2g\frac{(p_2 - p_1)}{\gamma}}$$

but,

$$p_1 - \gamma\ell - \gamma_m h + \gamma(\ell + h) = p_2 \quad \text{or} \quad p_2 - p_1 = (\gamma - \gamma_m)h$$

so that

$$V_1 = \sqrt{2g\left(1 - \frac{\gamma_m}{\gamma}\right)h} = \left[2\left(9.81\,\frac{m}{s^2}\right)\left(1 - \frac{900\,\frac{kg}{m^3}}{999\,\frac{kg}{m^3}}\right)(2.5\,m)\right]^{1/2}$$

$$= 2.20\,\frac{m}{s}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.08\,m)^2\left(2.20\,\frac{m}{s}\right) = \underline{\underline{0.0111\,\frac{m^3}{s}}}$$

3.70

3.70 The specific gravity of the manometer fluid shown in Fig. P3.70 is 1.07. Determine the volume flowrate, Q, if the flow is inviscid and incompressible and the flowing fluid is (a) water, (b) gasoline, or (c) air at standard conditions.

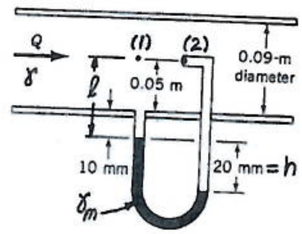


FIGURE P3.70

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } z_1=z_2 \text{ and } V_2=0$$

Thus,

$$V_1=\sqrt{2g\,\frac{(p_2-p_1)}{\gamma}} \qquad (1)$$

But

$$p_1+\gamma\ell+\gamma_m h=p_2+\gamma(\ell+h)$$

or

$p_2-p_1=(\gamma_m-\gamma)h$ so that Eq. (1) becomes

$$V_1=\sqrt{2g\,\frac{(\gamma_m-\gamma)}{\gamma}h}=\sqrt{2\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1.07\,(9.8\times10^3\,\frac{N}{m^3})}{\gamma}-1\right)(0.02\,m)}$$

Thus,

$$Q=A_1V_1=\frac{\pi}{4}D_1^2V_1=\frac{\pi}{4}(0.09\,m)^2\sqrt{2(9.81)\left(\frac{10.49\times10^3}{\gamma}-1\right)(0.02)}$$

or

$$Q=3.99\times10^{-3}\sqrt{\frac{10.49}{\gamma}-1}\ \frac{m^3}{s} \quad \text{where } \gamma\sim\frac{kN}{m^3}$$

For the given fluids this gives:

	fluid	$\gamma,\ \frac{kN}{m^3}$	$Q,\ \frac{m^3}{s}$
(a)	water	9.80	1.06×10^{-3}
(b)	gasoline	6.67	3.02×10^{-3}
(c)	air	12×10^{-3}	0.118

3.86

3.86 The vent on the tank shown in Fig. P3.86 is closed and the tank pressurized to increase the flowrate. What pressure, p_1, is needed to produce twice the flowrate of that when the vent is open?

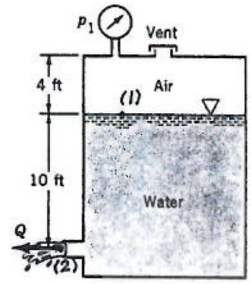


FIGURE P3.86

With the vent open:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \text{where } p_1=p_2=0,\ V_1=0,\ z_2=0$$

Thus,

$$z_1=\frac{V_2^2}{2g} \quad \text{or} \quad V_2=\sqrt{2gz_1}=\sqrt{2(32.2\,\tfrac{ft}{s^2})(10\,ft)}=25.4\,\tfrac{ft}{s}$$

To have double the flowrate with the vent closed ($p_1 \neq 0$):

$$\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g} \quad \text{where for this case } V_2=2(25.4\,\tfrac{ft}{s})=50.8\,\tfrac{ft}{s}$$

Thus,

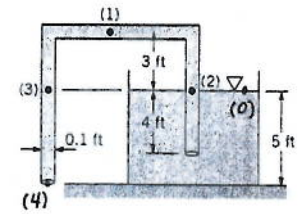
$$\frac{p_1}{(62.4\,\frac{lb}{ft^3})}+10\,ft=\frac{(50.8\,\frac{ft}{s})^2}{2(32.2\,\frac{ft}{s^2})}$$

or

$$p_1=1876\,\tfrac{lb}{ft^2}=\underline{\underline{13.0\,psi}}$$

3.87

3.87 Water is siphoned from the tank shown in Fig. P3.87. Determine the flowrate from the tank and the pressures at points (1), (2), and (3) if viscous effects are negligible.



■ FIGURE P3.87

From the Bernoulli equation,

$p_0 + \frac{1}{2}\rho V_0^2 + \gamma z_0 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4$, where $p_0 = p_4 = 0$, $V_0 = 0$, $z_0 = 5\,ft$, and $z_4 = 0$

Thus,

$\gamma z_0 = \frac{1}{2}\rho V_4^2$, or $V_4 = \sqrt{2\gamma z_0/\rho} = \sqrt{2 g z_0} = \sqrt{2(32.2\frac{ft}{s^2})(5 ft)} = 17.94\frac{ft}{s}$

Hence,

$Q = A_4 V_4 = \frac{\pi}{4}(0.1 ft)^2(17.94\frac{ft}{s}) = \underline{\underline{0.141\frac{ft^3}{s}}}$

For p_1: $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_1 = 8 ft$, and $V_1 = V_4$ (since $A_1 = A_4$) becomes

$p_1 = -\gamma z_1 = -(62.4\,lb/ft^3)(8 ft) = \underline{\underline{-499\frac{lb}{ft^2}}}$

For p_3: $p_3 + \frac{1}{2}\rho V_3^2 + \gamma z_3 = p_4 + \frac{1}{2}\rho V_4^2 + \gamma z_4$, which with $p_4 = 0$, $z_4 = 0$, $z_3 = 5 ft$, and $V_3 = V_4$ (since $A_3 = A_4$) becomes

$p_3 = -\gamma z_3 = -(62.4\,lb/ft^3)(5 ft) = \underline{\underline{-312\ lb/ft^2}}$

For p_2: Since $z_2 = z_3$ and $V_2 = V_3$ it follows that

$p_2 = p_3 = \underline{\underline{-312\ lb/ft^2}}$

3.113

3.113 Water flows in a rectangular channel that is 2.0 m wide as shown in Fig. P3.113. The upstream depth is 70 mm. The water surface rises 40 mm as it passes over a portion where the channel bottom rises 10 mm. If viscous effects are negligible, what is the flowrate?

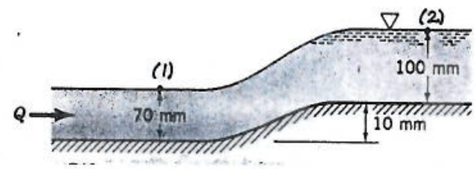


FIGURE P3.113

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_1 = 0,\ p_2 = 0,\ z_1 = 0.07\,m, \quad (1)$$

and $z_2 = (0.01 + 0.10)m = 0.11\,m$

Also, $A_1V_1 = A_2V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{0.07\,m}{0.10\,m} V_1 = 0.7V_1$$

Thus, Eq. (1) becomes

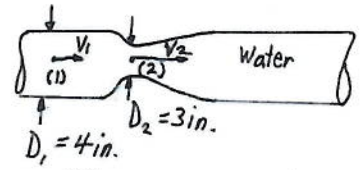
$$[1 - 0.7^2]V_1^2 = 2(9.81\tfrac{m}{s^2})(0.11 - 0.07)m \quad \text{or } V_1 = 1.24\tfrac{m}{s}$$

Hence,

$$Q = A_1V_1 = (0.07m)(2.0m)(1.24\tfrac{m}{s}) = \underline{\underline{0.174\ \tfrac{m^3}{s}}}$$

3.115

3.115 A Venturi meter with a minimum diameter of 3 in. is to be used to measure the flowrate of water through a 4-in.-diameter pipe. Determine the pressure difference indicated by the pressure gage attached to the flow meter if the flowrate is 0.5 ft³/s and viscous effects are negligible.



$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}} \text{, where } Q = 0.5\,\frac{ft^3}{s} \text{ and } \rho = 1.94\,\frac{slugs}{ft^3}$$

Thus, since $A_2/A_1 = (D_2/D_1)^2$,

$$0.5\,\frac{ft^3}{s} = \frac{\pi}{4}\left(\frac{3}{12}\,ft\right)^2 \sqrt{\frac{2(p_1 - p_2)}{(1.94\,\frac{slugs}{ft^3})[1-(3in./4in.)^4]}}$$

or

$$p_1 - p_2 = 68.8\,\frac{slug}{s^2\,ft} = 68.8\left(\frac{slug \cdot ft}{s^2}\right)/ft^2 = \underline{\underline{68.8\,\frac{lb}{ft^2}}}$$

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3.116

3.116 Determine the flowrate through the Venturi meter shown in Fig. P3.116 if ideal conditions exist.

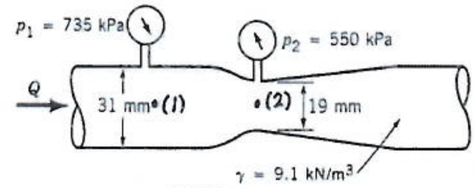

FIGURE P3.116

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$ where $z_1=z_2$ and $A_1V_1=A_2V_2$

or

$$V_1=\frac{A_2}{A_1}V_2=\left(\frac{D_2}{D_1}\right)^2V_2$$

Thus,

$$\frac{p_1}{\gamma}+\frac{\left(\frac{D_2}{D_1}\right)^4V_2^2}{2g}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}$$

or

$$V_2=\sqrt{\frac{2g\,\frac{(p_1-p_2)}{\gamma}}{1-\left(\frac{D_2}{D_1}\right)^4}}=\sqrt{\frac{2\left(9.81\,\frac{m}{s^2}\right)\frac{(735-550)\,kPa}{\left(9.1\,\frac{kN}{m^3}\right)}}{1-\left(\frac{19\text{ mm}}{31\text{ mm}}\right)^4}}=21.5\,\frac{m}{s}$$

so that

$$Q=A_2V_2=\frac{\pi}{4}D_2^2V_2=\frac{\pi}{4}(0.019\,m)^2\left(21.5\,\frac{m}{s}\right)=\underline{\underline{6.10\times10^{-3}\,\frac{m^3}{s}}}$$